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COLLISIONAL PHASE SHIFTS BETWEEN TWO DUST ACOUSTIC SOLITARY WAVES WITH EXTERNAL OBLIQUE MAGNETIC FIELD AND TWO TEMPERATURE IONS

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ABSTRACT

Using the extended Poincaré-Lighthill-Kuo (PLK) method, a theoretical investigation for nonlinear electrostatic dust acoustic solitary waves (DASWs) with external oblique magnetic field and two temperature ions is studied. The interesting feature of the nonlinear evolution equations for the DASWs is two Korteweg-de Vries (KdV) equations for small but finite amplitude solitary waves. The phase shifts and trajectories of DASWs after collision are deduced. Results in summary insure the effects of some physical parameters such as the obliqueness angle of the external magnetic field, the effective temperature of ions to the electron temperature ratio, and the two-different types of ions on the phase shifts for both the physical process (isothermal and adiabatic). Colors online provided strong effects for these factors on the wave future.

1. INTRODUCTION

Two decade ago, the existence of dust acoustic solitary waves (DASWs) which are new normal mode of dusty plasma (plasma with extremely massive and highly negatively charged dust grains) has been reported [1]. These waves observed in a laboratory experiment [2]. From then on, a great number of experimental investigations [3-5] and a large number of theoretical studies [6-13] have been made to examine the features of DASWs in dusty plasmas which has taken the key position in space and laboratory plasma. Dust acoustic solitary waves in a magnetized dusty plasma consisting of a negatively charged dust fluid and Boltzmann distribution for electrons and ions has studied [14]. He demonstrated that, DASWs have large amplitude, small width, and high propagation velocity. A great effort has been done to study nonlinear properties of solitary waves in a magnetized dusty plasma consisting of non isothermal ions, isothermal electrons and variable charged dust grains [15]. The results of this model showed dependence for the wave amplitude and width on the dust charge variation. Dust particles can be charged either positively or negatively due to a variety of processes including the bombardment of the dust grains surface by background plasma electrons, ions, photoelectron emission by ultraviolet (uv) radiation, ion sputtering, secondary electron production [16-18]. For example in low-temperature laboratory dusty plasma dust particles usually acquire a negative charge because the thermal speed of the electrons is much higher than that of the ions. In the normal laboratory condition, these charged particles levitate in the sheath region where strong electric field exists. The direct effect of this electric field is to exert electric force on the particle which balances the gravity. On the other hand the indirect effect is to produce ion and electron drag forces on the grains. The competitions between these different types of forces are responsible to determine different types of static and dynamic properties of grains, affect wave phenomena etc [19]. On the other side, two-temperature ions assumption also has been discussed in many plasma researches [20-22]. This assumption arises due to the relative difference in the kinetic energy between two types of ions for the gas introduced with different masses of ions (leads to different velocities) which make one type with higher temperature than the other. Accordingly, the study of small amplitude DASWs in the existence of two temperature isothermal ions [20]. He showed that both compressive and rarefactive solitary waves are existing. The energy rate between the two types of ions must be larger than the characteristic frequency ω_{pd} where $\omega_{pd}=(4\pi n_{do}e^2z_d^2/m_d)^{1/2}$ [n_{do} is the unperturbed dust density, z_d is the dust charge number, and m_d is the dust mass] to achieve validity of two temperature ion assumption in a small and large amplitude DASWs in dusty plasma was illustrated [21]. A valuable study on the interaction between two DASWs in a magnetized dusty plasma and two temperature isothermal ions presented [22]. The results of this model showed a significant effect of some parameters (concentration of electrons, two-type of isothermal ions, and adiabatic ratio of specific heats) on the wave propagation after collision. We known that: when two solitary waves approaches closely they interact, exchange their energies and positions and then separate off achieving a unique effect which is their phase shift [23]. Recently, the investigation of DIAWs in a four component magnetized dusty plasma has been carried out [24] where the Zakharov-Kuznetsov equation is derived using reductive perturbation technique on the basis of two fluid model. In 1-D system, solitary waves can interact with two different ways. The overtaking collision at which the angle between two propagation directions of two solitary waves equals zero and we can study this by using the inverse scattering transformation method [25]. The head-on collision at which the angle between two propagation directions of two solitary waves equals π and many authors [26-27] use the famous Poincaré-Lighthill-Kuo (PLK) method to solve their model original fluid-dynamic equations. For example, the head-on collision between two DASWs in a magnetized dusty plasma has investigated [8]. The author of this reference found that the magnitude and the obliqueness of the external magnetic field have strong effects on the phase shifts. To our knowledge; a few number of scholar use an isothermal and adiabatic descriptions of formation and dynamics on nonlinear waves occur in dusty plasma. The isothermal assumption for the thermal process means absorption of energy upon compression and releasing it upon rarefaction [7]. The adiabatic approach enables one to include the variation of temperature for various phases of the wave and the effect of this variation on the formation and properties of the wave proper [28]. Accordingly: the isothermal and the adiabatic approaches are considered as a realistic description which have the same probability of occurrence and can be controlled in plasma laboratory (Tokamakes). For example, the head-on collision of two electrostatic solitary waves (ESWs) in electron-positron-ion plasma studied [29]. He showed that the ion-to-electron number density ratio and positron-to-electron temperature ratio have a strong effect on the phase shifts for both of the physical process (isothermal and adiabatic). In this paper, by using the extended PLK method, we study the head-on collision between two DASWs with external oblique magnetic field and two-temperature isothermal ions. The phase shifts and trajectories of DASWs after collision are deduced. Also, results in summary showed the effects of some physical parameters such as obliqueness angle of the external magnetic field and the two-different type of ions on the phase shift for both of the physical process. The plan of this paper is as follows: the basic equations governing the plasma system under consideration is presented, the Korteweg - de Vries (KdV) equations, and the analytical phase shifts and trajectories after the head-on collision is derived in section 2. Discussion and Conclusion are given in section 3

2. BASIC EQUATIONS AND INTERACTION BETWEEN TWO DASWS

Consider a magnetized dusty plasma with either adiabatic or isothermal dust grains, two distinct groups of isothermal ions one with a lower temperature T_{il}, the other acquires a higher temperature T_{ih} and having densities n_{il} and n_{ih}, respectively, beside isothermal electrons. In the x-z plane, an external static magnetic field **B**= ($B_0 \cos\theta\ \hat{x}$ +$B_0 \sin\theta\ \hat{z}$) was applied, making an angle θ with the wave propagation vector x-axis. B_0 is the magnitude of the magnetic field. Physically, in the DASWs, the restoring force comes from the electron–ion pressure while the dust mass provides the inertia to maintain the DASWs. Thus, the two-temperature isothermal ions number densities are given by [30]:

$$n_{il} = \mu_{il} \exp(\sigma_{il} \phi) \tag{1}$$

$$n_{ih} = \mu_{ih} \exp(\sigma_{ih} \phi) \tag{2}$$

while for isothermal electrons

$$n_e = \mu_e \exp(\sigma_e \phi) \tag{3}$$

where $\mu_{il} = n_{il0} / Z_d n_{d0}$, $\mu_{ih} = n_{ih0} / Z_d n_{d0}$, $\mu_e = n_{e0} / Z_d n_{do}$, $\sigma_{il} = T_{eff} / T_{il}$, $\sigma_{ih} = T_{eff} / T_{ih}$, $\sigma_e = T_{eff} / T_e$, $\sigma_d = T_d / T_{eff}$, $T_{eff} = T_{il}(1+\mu_e)/(\mu_{il} + \mu_{ih}\sigma_{lh})$ with $\sigma_{lh} = T_{il} / T_{ih}$ and n_{il}, n_{ih}, n_e, n_d and ϕ are the densities of low, high temperature ions, electrons, dust grains (which are normalized by $Z_d n_{d0}$), and the electrostatic potential (normalized by $Z_d^2 T_d n_{d0}$), respectively. The dynamics of DASWs are governed by the following equations [7].

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x} n_d u_{dx} = 0, \tag{4}$$

$$\frac{\partial u_{dx}}{\partial t} + u_{dx} \frac{\partial}{\partial x} u_{dx} = (\Delta Q - 1)\left(-\frac{\partial}{\partial x}\phi + \omega_{cd} v_{dy} \sin\theta \right) - \frac{\sigma_d}{n_d} \frac{\partial p_d}{\partial x} \tag{5}$$

,

$$\frac{\partial v_{dy}}{\partial t} + u_{dx} \frac{\partial}{\partial x} v_{dy} = -\omega_{cd} (\Delta Q - 1)(u_{dx} \sin\theta - w_{dz} \cos\theta), \tag{6}$$

$$\frac{\partial w_{dz}}{\partial t} + u_{dx} \frac{\partial}{\partial x} w_{dz} = -\omega_{cd} (\Delta Q - 1) v_{dy} \cos\theta, \tag{7}$$

$$\frac{\partial p_d}{\partial t} + \gamma p_d \frac{\partial}{\partial x} u_{dx} = 0, \tag{8}$$

The system of equations is closed by Poisson equation

$$\frac{\partial^2 \phi}{\partial x^2} = -(\Delta Q - 1)\, n_d + n_e - n_{il} - n_{ih}\,, \tag{9}$$

and the charging equation

$$\frac{\partial Q_d}{\partial t} = I_{il} + I_{ih} + I_e\,. \tag{10}$$

In the present model we define some important parameters such as: the specific heat ratio $\gamma=C_p/C_v$ which indicates $\gamma=1$for the isothermal case and $\gamma=3$ for the adiabatic case, Q_d is the dust charge [$=-Z_d e + \Delta Q_d$], where ΔQ_d and Z_d are the variation and the number of charges on the dust grain, and we have $(\Delta Q - 1)\,[\Delta Q = \Delta Q_d / e Z_d]$ normalized by $Z_d e$. Also: u_{dx}, v_{dy}, and w_{dz} are the dust velocities in x, y, z directions which are normalized by T_{eff}/e. In eqs.4-10, the space coordinate x, plasma pressure p_d and time variable t are normalized by Debye length $\lambda_{dD}=(T_{eff}/4\pi n_{d0} Z_d e^2)^{1/2}$, $(Z_d T_{eff}/m_d)^{1/2}$, and the inverse of dust plasma frequency ω_{pd}^{-1} respectively. The dust cyclotron frequency ω_{cd} is normalized by ω_{pd} where $\omega_{cd}=(B_0 e Z_d / c\, m_d)/\omega_{pd}$, e is the magnitude of the electron charge. With the aid of formula for the electrons and ions currents [31]: at equilibrium $I_e + I_{il} + I_{ih} \approx 0$ [31, 32] eq. 10 can written in the form

$$\left[-\beta_1 n_e e^{\left(Z_1 (\Delta Q - 1)\right)} + n_{il}\left(1 - Z\ \ (\Delta Q - 1)\right) + \beta_2 n_{ih}\left(1 - Z_2 (\Delta Q - 1)\right)\right] = 0 \tag{11}$$

where

$$Z = \left(\frac{Z_d e^2}{r_d T_{il}}\right),\ Z_1 = \left(\frac{Z \sigma_e}{\sigma_{il}}\right),\ Z_2 = Z \sigma_{lh}\,,\ \beta_1 = \left(\frac{\sigma_{il}}{16 \sigma_e \mu_{ei}}\right)^{1/2},\ \mu_{ei} = \frac{m_e}{m_i}\,,\ \beta_2 = \left(\frac{1}{\sigma_{lh}}\right)^{1/2}.$$

m_e, m_i and r_d are the electron mass, the ion mass, and the dust grain radius, respectively. The charge neutrality at equilibrium requires, $Z_i n_{i0} = Z_i (n_{il0} + n_{ih0}) = n_{e0} + Z_d n_{d0}$, where, n_{d0}, n_{e0} and n_{i0}($=n_{il0} + n_{ih0}$) are the unperturbed dust density, electron density, and ion number densities at the two different temperatures T_{il} and T_{ih}. Z_i is the number of charge on the ion and $Z_i= 1$ for a singly ionized plasma system.

Assume two solitons S_1 and S_2 having small amplitudes~ε (where ε is a smallness formal perturbation parameter characterizing the strength of nonlinearity) which are asymptotically, far apart in the initial state and travel towards each other. When they become close, they interact, exchange their energies and positions with each other and then separate off, regaining their original wave forms except the phase shift (weak collision). Using the PLK perturbation method [26], we study the effects of this quasielastic collision. According to this method, the dependent variables are expanded as [26, 27].

$$\psi = \psi^{(0)} + \sum_{n=1}^{\infty} \varepsilon^{(n+1)} \psi^{(n)}, \tag{12}$$

with

$$\psi = [n_d, u_{dx}, v_{dy}, w_{dz}, \phi, \Delta Q, p_d], \tag{13}$$

and

$$\psi^{(0)} = [1,0,0,0,0,0,1], \tag{14}$$

The stretched coordinates are given as follows:

$$\xi = \varepsilon(x - \lambda t) + \varepsilon^2 P^{(0)}(\eta, \tau) + \varepsilon^3 P^{(1)}(\xi, \eta, \tau) + \ldots, \tag{15}$$

$$\eta = \varepsilon(x + \lambda t) + \varepsilon^2 Q^{(0)}(\eta, \tau) + \varepsilon^3 Q^{(1)}(\xi, \eta, \tau) + \ldots, \tag{16}$$

$$\tau = \varepsilon^3 t, \tag{17}$$

where ζ and η denote the trajectories of the two propagating solitary waves, respectively, and λ is the unknown phase velocity of DASWs. The parameters λ, $P^{(0)}(\eta,\tau)$ and $Q^{(0)}(\xi,\tau)$ are to be determined later. Substituting eqs.12- 17 into eqs.1- 10 and equating terms of like powers of ε, we obtain coupled equations in different orders of ε. To the leading order, we have

$$\lambda\left(-\frac{\partial}{\partial\xi} + \frac{\partial}{\partial\eta}\right)n_1 + \left(\frac{\partial}{\partial\xi} + \frac{\partial}{\partial\eta}\right)u_1 = 0, \tag{18}$$

$$\lambda\left(-\frac{\partial}{\partial\xi} + \frac{\partial}{\partial\eta}\right)u_1 = \left(\frac{\partial}{\partial\xi} + \frac{\partial}{\partial\eta}\right)\phi_1 - \omega_{cd} v_1 \sin\theta - \sigma_d\left(\frac{\partial}{\partial\xi} + \frac{\partial}{\partial\eta}\right)p_1, \tag{19}$$

$$-\omega_{cd}\omega_1 \cos\theta + \omega_{cd} u_1 \sin\theta = 0, \tag{20}$$

$$\lambda\left(-\frac{\partial}{\partial\xi} + \frac{\partial}{\partial\eta}\right)p_1 + \gamma\left(\frac{\partial}{\partial\xi} + \frac{\partial}{\partial\eta}\right)u_1 = 0, \tag{21}$$

$$\lambda\left(-\frac{\partial}{\partial\xi} + \frac{\partial}{\partial\eta}\right)w_1 - \omega_{cd} v_1 \cos\theta = 0, \tag{22}$$

$$\mu_{il} + \mu_{ih} - \mu_e = 1. \tag{23}$$

The charging equation gives:

$$-[\sigma_e\Omega_2+\mu_{il}\sigma_{il}(1+Z)+\beta_2\mu_{ih}\sigma_{ih}(1+Z_2)]\phi_1=(\beta_2 Z_2\mu_{ih}+\mu_{il}Z+Z_1\Omega_2)\Delta Q_1. \tag{24}$$

The solutions of eqs.18-24 read

$$\phi_1=\phi_1^{(1)}(\xi,\tau)+\phi_1^{(2)}(\eta,\tau), \tag{25}$$

$$n_1=\frac{-1}{(\lambda^2\sec^2\theta-\gamma\sigma_d)}[\phi_1^{(1)}(\xi,\tau)+\phi_1^{(2)}(\eta,\tau)], \tag{26}$$

$$u_1=-\frac{\lambda}{(\lambda^2\sec^2\theta-\gamma\sigma_d)}[\phi_1^{(1)}(\xi,\tau)-\phi_1^{(2)}(\eta,\tau)], \tag{27}$$

$$p_1=-\frac{\gamma}{(\lambda^2\sec^2\theta-\gamma\sigma_d)}[\phi_1^{(1)}(\xi,\tau)+\phi_1^{(2)}(\eta,\tau)], \tag{28}$$

$$v_1=\frac{\lambda^2\tan\theta}{\omega_{cd}\cos\theta(\lambda^2\sec^2\theta-\gamma\sigma_d)}[\frac{\partial\phi_1^{(1)}(\xi,\tau)}{\partial\xi}+\frac{\partial\phi_1^{(2)}(\eta,\tau)}{\partial\eta}], \tag{29}$$

$$w_1=\frac{-\lambda\tan\theta}{(\lambda^2\sec^2\theta-\gamma\sigma_d)}[\phi_1^{(1)}(\xi,\tau)-\phi_1^{(2)}(\eta,\tau)]\ , \tag{30}$$

$$\Delta Q_1=-\Omega_1[\phi_1^{(1)}(\xi,\tau)+\phi_1^{(2)}(\eta,\tau)], \tag{31}$$

where

$$\Omega_1=\left(\sigma_e\Omega_2+\mu_{il}\sigma_{il}(1+Z)+\beta_2\mu_{ih}\sigma_{ih}(1+Z_2)\right)/(\beta_2 Z_2\mu_{ih}+\mu_{il}Z+Z_1\Omega_2),$$

$$\Omega_2=\beta_2\mu_{ih}(1+Z_2)+\mu_{il}(1+Z),$$

$$\Omega_3=\frac{(\Omega_2\sigma_e^2-\beta_2\mu_{ih}\sigma_{ih}^2(1+Z_2)-\mu_{il}\sigma_{il}^2(1+Z)+\Omega_2 Z_1^2\Omega_1^2-2\sigma_e Z_1\Omega_1\Omega_2+2\beta_2\mu_{ih}\sigma_{ih}Z_2\Omega_1+2\mu_{il}\sigma_{il}Z\Omega_1)}{2(\Omega_2 Z_1+\beta_2\mu_{ih}Z_2+\mu_{il}\sigma_{il})},$$

with the condition to obtain a uniquely defined n_1, u_1, υ_1, ω_1, $p_1$and Q_1 from eqs.25-31, the wave velocity is given by

$$\lambda=\sqrt{\frac{\gamma\sigma_d}{\sec^2\theta}+\frac{\cos^2\theta}{\mu_{il}\sigma_{il}+\mu_{ih}\sigma_{ih}+\mu_e\sigma_e+\Omega_1}} \tag{32}$$

The unknown functions $\phi_1^{(1)}(\xi,\tau)$, and $\phi_1^{(2)}(\eta,\tau)$, will be determined from the next orders in ε. Equations 25-31 imply that at the leading order, we have two waves, one of which $\phi_1^{(1)}(\xi,\tau)$, is traveling to right, and the other one $\phi_1^{(2)}(\eta,\tau)$, is traveling to left. Furthermore, after some algebraic steps we can obtain an equation as (see [9])

$$\begin{aligned}
2\left(\lambda^2 \sec^2\theta - \gamma\sigma_d\right)u_3 &= \int\left(\frac{\partial \phi_1^{(1)}(\xi,\tau)}{\partial\tau} - A_1\phi_1(\xi,\tau)\frac{\partial \phi_1^{(1)}(\xi,\tau)}{\partial\xi} + B_1\frac{\partial^3 \phi_1^{(1)}(\xi,\tau)}{\partial\xi^3}\right)d\eta \\
&+ \int\left(\frac{\partial \phi_1^{(2)}(\eta,\tau)}{\partial\tau} + A_1\phi_1^{(1)}(\eta,\tau)\frac{\partial \phi_1^{(2)}(\eta,\tau)}{\partial\eta} - B_1\frac{\partial^3 \phi_1^{(2)}(\eta,\tau)}{\partial\eta^3}\right)d\xi \\
&+ \iint\left(A_2\frac{\partial p^{(0)}(\eta,\tau)}{\partial\eta} - B_2\phi_1^{(2)}(\eta,\tau)\right)\frac{\partial^2 \phi_1^{(1)}(\xi,\tau)}{\partial\xi^2}d\xi d\eta \\
&- \iint\left(A_2\frac{\partial Q^{(0)}(\xi,\tau)}{\partial\xi} - B_2\phi_1^{(1)}(\xi,\tau)\right)\frac{\partial^2 \phi_1^{(2)}(\eta,\tau)}{\partial\eta^2}d\xi d\eta,
\end{aligned} \tag{33}$$

where

$$A_1 = -\frac{\alpha_2^2\cos^2\theta\left(3\Omega_1 - \alpha_2\alpha_1\right) - \gamma\sigma_d\cos^2\theta\left(\gamma-3\right) - 3\lambda^2}{2\lambda\alpha_2},$$

$$B_1 = \frac{\lambda^4\tan^2\theta + \omega_{cd}^2\alpha_2^2\cos^2\theta}{2\,\omega_{cd}^2\lambda}, \quad A_2 = 2\lambda,$$

$$B_2 = \frac{\alpha_2^2\cos^2\theta\left(\alpha_1\alpha_2 - 3\Omega_1\right) + \gamma\sigma_d\cos^2\theta\left(\gamma-1\right) - \lambda^2}{2\lambda\alpha_2},$$

$\alpha_1 = \left(2\Omega_3 + \mu_e\,\sigma_e^2 - \mu_{il}\,\sigma_{il}^2 - \mu_{ih}\,\sigma_{ih}^2\right)$, and $\alpha_2 = (\lambda^2\sec^2\theta - \gamma\sigma_d)$.

The first-(second) term on the right hand side of eq. 33 are all secular terms(proportional to $\eta(\xi)$ and the integrated function is independent of $\eta(\xi)$) which must be eliminated in order to avoid spurious resonances and the third and fourth terms are not secular terms in this order, but they could be secular in the next order. Hence we have

$$\frac{\partial \phi_1^{(1)}(\xi,\tau)}{\partial\tau} - A_1\,\phi_1^{(1)}(\xi,\tau)\frac{\partial \phi_1^{(1)}(\xi,\tau)}{\partial\xi} + B_1\frac{\partial^3 \phi_1^{(1)}(\xi,\tau)}{\partial\xi^3} = 0, \tag{34}$$

$$\frac{\partial \phi_1^{(2)}(\eta,\tau)}{\partial\tau} + A_1\,\phi_1^{(2)}(\eta,\tau)\frac{\partial \phi_1^{(2)}(\eta,\tau)}{\partial\eta} - B_1\frac{\partial^3 \phi_1^{(2)}(\eta,\tau)}{\partial\eta^3} = 0. \tag{35}$$

$$\frac{\partial p^{(0)}(\eta,\tau)}{\partial\eta} - \frac{B_2}{A_2}\phi_1^{(2)}(\eta,\tau) = 0, \tag{36}$$

$$\frac{\partial Q^{(0)}(\xi,\tau)}{\partial \xi}-\frac{B_2}{A_2}\phi_1^{(1)}(\xi,\tau)=0\,, \tag{37}$$

Equations 34 and 35 are the two-side traveling KdV equations in the reference frames ξ and η, respectively. Their solutions are [12]

$$\phi_1^{(1)}(\xi,\tau)=\phi_{s_1}\operatorname{sech}^2\left[\left(\frac{A_1\phi_{s_1}}{12B_1}\right)^{1/2}\left(\xi-\frac{1}{3}A_1\phi_{s_1}\tau\right)\right], \tag{38}$$

$$\phi_1^{(2)}(\eta,\tau)=\phi_{s_2}\operatorname{sech}^2\left[\left(\frac{A_1\phi_{s_2}}{12B_1}\right)^{1/2}\left(\eta+\frac{1}{3}A_1\phi_{s_2}\tau\right)\right], \tag{39}$$

where ϕ_{s_1} and ϕ_{s_2} are the amplitude of the two solitons S_1 and S_2 in their initial positions. The leading phase changes due to the collision can be calculated from eqs. 36 and 37 as

$$p^{(0)}(\eta,\tau)=-\frac{B_2}{A_2}\left(\frac{12B_1\phi_{s_2}}{A_1}\right)^{1/2}\left\{\tanh\left[\left(\frac{A_1\phi_{s_2}}{12b_1}\right)^{1/2}\left(\eta+\frac{1}{3}A_1\phi_{s_2}\tau\right)\right]-1\right\} \tag{40}$$

,

$$Q^{(0)}(\xi,\tau)=-\frac{B_2}{A_2}\left(\frac{12B_1\phi_{s_1}}{A_1}\right)^{1/2}\left\{\tanh\left[\left(\frac{A_1\phi_{s_1}}{12B_1}\right)^{1/2}\left(\xi-\frac{1}{3}A_1\phi_{s_1}\tau\right)\right]+1\right\}. \tag{41}$$

Hence, up to $o(\varepsilon^2)$, the trajectories of the two solitary waves for a weak head-on interaction are:

$$\xi=\varepsilon(x-\lambda t)-\varepsilon^2\frac{B_2}{A_2}\left(\frac{12B_1\phi_{s_2}}{A_1}\right)^{1/2}\left\{\tanh\left[\left(\frac{A_1\phi_{s_2}}{12B_1}\right)^{1/2}\left(\eta+\frac{1}{3}A_1\phi_{s_2}\tau\right)\right]-1\right\}+\ldots, \tag{42}$$

$$\eta=\varepsilon(x+\lambda t)-\varepsilon^2\frac{B_2}{A_2}\left(\frac{12B_1\phi_{s_1}}{A_1}\right)^{1/2}\left\{\tanh\left[\left(\frac{A_1\phi_{s_1}}{12B_1}\right)^{1/2}\left(\xi-\frac{1}{3}A_1\phi_{s_1}\tau\right)\right]+1\right\}+\ldots. \tag{43}$$

To obtain the phase shifts after a head-on collision of the two solitons, we assume that the two solitons S_1 and S_2 are, asymptotically, far from each other at the initial time $(t=-\infty)$, i.e., soliton S_1 is at $\xi=0,\eta=-\infty$ and soliton S_2 is at $\eta=0,\xi=+\infty$. After the collision $(t = +\infty)$, the soliton S_1 is far to the right of soliton S_2, i.e., soliton S_1 is at $\xi = 0$, $\eta = +\infty$ and soliton S_2 is at $\eta = 0, \xi = -\infty$ .Using eqs. 42 and 43 we obtain the corresponding phase shifts $\Delta P^{(0)}$ and $\Delta Q^{(0)}$ as follows [9]

$$\Delta P^{(0)} = -2\varepsilon^2 \frac{B_2}{A_2}\left(\frac{12B_1\phi_{s_2}}{A_1}\right)^{1/2}, \tag{44}$$

$$\Delta Q^{(0)} = 2\varepsilon^2 \frac{B_2}{A_2}\left(\frac{12B_1\phi_{s_1}}{A_1}\right)^{1/2}. \tag{45}$$

3. DISCUSSION AND CONCLUSION

In the present work, we investigated the head-on collision between two DASWs for a system consisting of negatively charged dust particles (adiabatic or isothermal), external oblique magnetic field, isothermal electrons, and two temperature isothermal ions using the extended PLK method. Since soliton S_1 is traveling to the right and soliton S_2 is traveling to the left, we see from eqs. 44 and 45 that due to collision each soliton has a negative (positive) phase shift for the adiabatic (isothermal) process in its traveling direction. The negative phase shift after the DASWs collision means that the velocities of the propagated DASWs are on average faster than that of the DASWs before the interactions [33]. The positive phase shift indicates that the postcollision part of the solitary waves moves ahead of the initial trajectory [34]. The magnitudes of the phase shift are related to the physical parameters $\varepsilon, \sigma_{lh}, \sigma_d, \omega_{cd}, \mu_{il}, \mu_{ih}$ and θ. For investigating the effects of various physical parameters on the phase shifts, we assume $\varepsilon = 0.1$, $\phi_{s_1} = \phi_{s_2} = 0.1$. Color online showed how the system parameters affect the phase shifts for both of the adiabatic and the isothermal processes. Figure 1 a(b) shows the variation of the negative (positive) phase shift $\Delta Q^{(0)}$ ($\Delta P^{(0)}$)for adiabatic (isothermal) case with the effective temperature of ions to the electron temperature ratio σ_e (T_{eff}/T_e) for different values of the dust temperature to the ions effective temperature ratio σ_d (T_d/T_{eff}). The two figures indicate that the phase shift $\Delta Q^{(0)}$ ($\Delta P^{(0)}$) increases with increasing both of σ_e and σ_d for a given value of σ_d and σ_e ,respectively. The phase shift curves are away from each other with increasing σ_e and σ_d, which means that these two parameters play a circular role on the variation of the phase shifts.

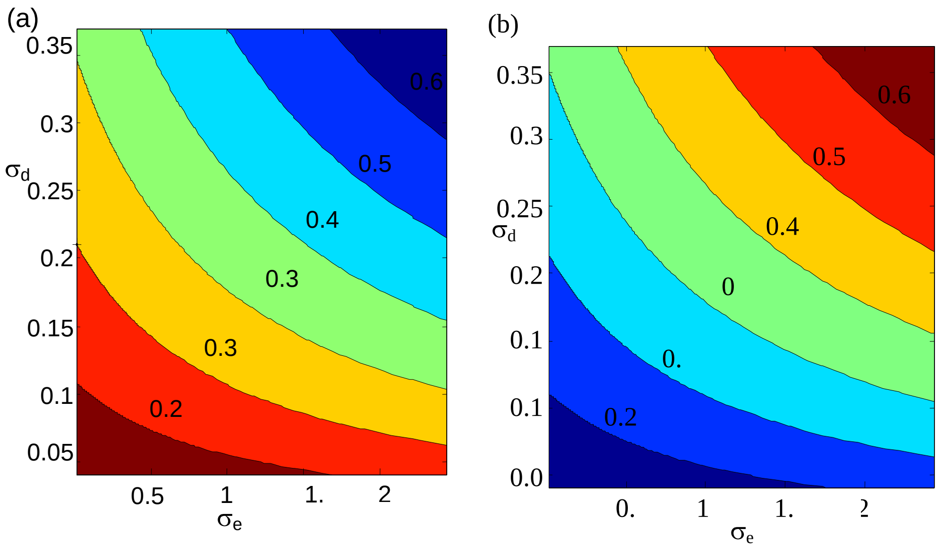

Figure 1a(b). Variation of $\Delta Q^{(0)}$ in the $\sigma_e - \sigma_d$ plane, where the numbers on the contour lines indicate the values of the corresponding phase shift $\Delta Q^{(0)}$, for $\gamma = 3$ (1), $\omega_{cd} = 0.0707, \theta = 45$.

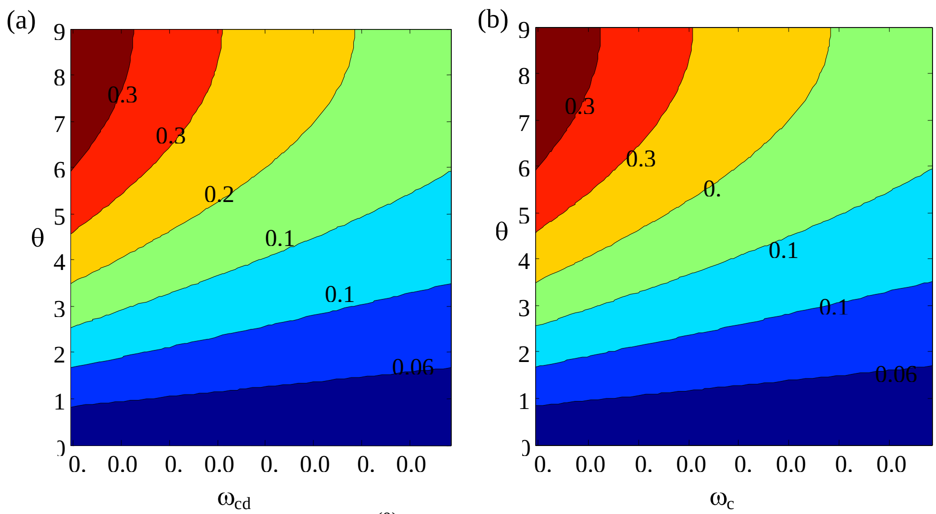

Figure 2a(b). Variation of $\Delta Q^{(0)}$ in the $\omega_{cd} - \theta$ plane, where the numbers on the contour lines indicate the values of the corresponding phase shift $\Delta Q^{(0)}$, for $\gamma = 3(1)$, $\sigma_{lh} = 0.3$.

On the other side, Fig. 2 a(b) represents the variation of the phase shifts($\Delta Q^{(0)}$ or $\Delta P^{(0)}$) with ω_{cd} and θ. It is clear that the phase shift increases as θ increases but decreases as ω_{cd} increases. For a given value of ω_{cd}, the phase shift increases rapidly with θ for $\theta < 60^o$ and increases smoothly with θ for $\theta > 60^o$. On one hand, for a given value of θ, the phase shift curves are far away from each other for ω_{cd} values. This indicates that both θ and ω_{cd} represent the key parameter in the soliton collision which agrees with [7].

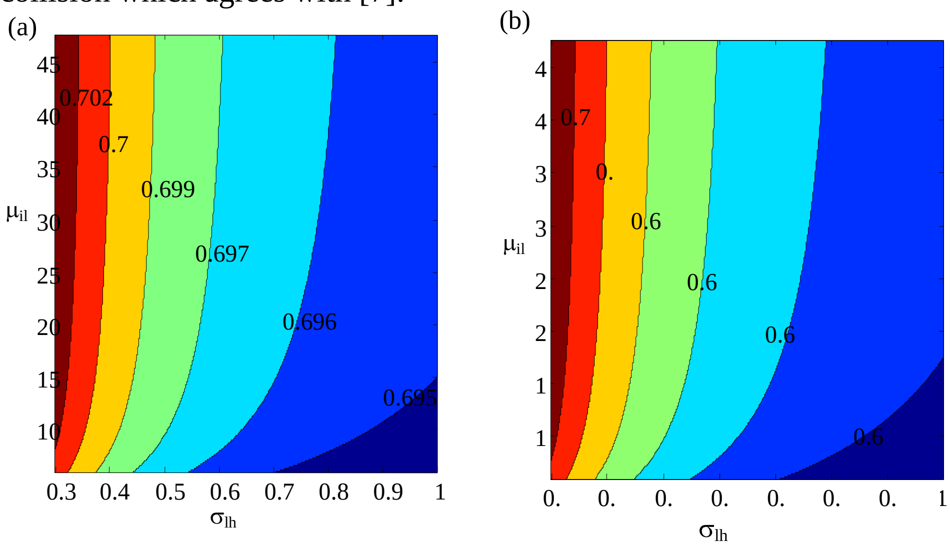

Figure 3a(b). Variation of $\Delta Q^{(0)}$ in the $\sigma_{lh} - \mu_{il}$ plane, where the numbers on the contour lines indicate the values of the corresponding phase shift $\Delta Q^{(0)}$, for $\gamma = 3(1)$, $\theta = 90$.

It is seen from Fig. 3 a(b) contour lines that the phase shift increases with $\mu_{il}(= n_{il0}/Z_d n_{d0})$ and decreases with $\sigma_{lh}(=T_{il}/T_{ih})$. For small (large) value of μ_{il}, the phase shift increases rapidly (smoothly) with the increase of σ_{lh}. Also, for a given value of μ_{il}, the phase shift curves are near (far) from each other for small (large) values of σ_{lh}. Accordingly, μ_{il} and σ_{lh} has a strong effect on the phase shifts.

To summarize, the phase shifts and the trajectories describing the head-on collision of two DASWs in a plasma composed of variable negatively charged dust grains, isothermal electrons and two-temperature isothermal ions in the presence of an external oblique magnetic field are investigated by the extended PLK method. The following interesting features are deduced.

1- The magnitude of the phase shift of the DASWs is directly depending on the physical parameters i.e. $\sigma_{lh}, \sigma_d, \omega_{cd}, \mu_{il}, \theta$ and σ_e.
2- The physical processes, either isothermal or adiabatic, play an important role on the future of the DASWs after collision.

Finally, it may pointed out hear that the results of the present model should be useful in understanding the collective phenomena related to DASWs collision for the isothermal and the adiabatic approaches which have the same probability of occurrence and can be controlled in plasma laboratory.

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ازاحة الطور الناشئة عن التصادم بين موجتان سلوتينيتان غباريتان صوتيتان في بلازما تحتوي علي مجال مغناطيسي مائل ونوعان مختلفان في درجة الحرارة من الالكترونات

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تم دراسة ازاحة الطور الناتجة عن التصادم بين موجتان سلوتينيتان غباريتان صوتيتان في بلازما تحتوي علي مجال مغناطيسي مائل ونوعان مختلفان في درجة الحرارة من الالكترونات وذلك باستخدام طريقة (Poincaré–Lighthill–Kuo) للحصول علي معادلة Korteweg–de Vries . من خلال الحل التقريبي لهذه المعادلة تم دراسة تاثير بعض المتغيرات علي هذا التصادم مثل زاوية ميل المجال المغناطيسي والنسبه بين درجه حرارة الالكترونات الي الايونات والنسبه بين درجة حرارة نوعي الالكترونات علي التاثير علي ازاحة الطور اثناء التصادم في الحالتين الايزوثرماليه والادياباتيكيه والمقارنه بينهم وقد وجد ان:

1- تعتمد ازاحة الطور بشكل اساسي علي بعض المتغيرات مثل $(\sigma_{lh}, \sigma_d, \omega_{cd}, \mu_{il}, \theta)$

2- حاله النظام الايزوثرماليه او الادياباتيكيه تؤثر بشكل فعال علي ازاحة الطور

لذلك فان هذا البحث علي درجه كبيرة من الاهمية لانه يبين ان هناك اختلاف واضح بين التصادم للموجات السلوتينية الغباريه بين الحالتين الايزوثرمالية و الاديباتيكية و هو مالم يدرس من قبل.